Revised WI Model Academic Standards for Mathematics – Grades PK-2

Instructional programs in prekindergarten through grade two should address the three model academic standards—concepts and connections in number and algebra, concepts and connections in geometry and measurement, and concepts and connections in data analysis and probability. It is essential that these standards be embedded in contexts that promote and develop essential mathematical process skills—problem solving, conjecture, reasoning, and proof, representation and visualization, communication and reflection, and connections.

PK-12 Standard: Concepts and Connections in Number and Algebra

Grades PK-2: Foundations of Number Sense *(Number, Algebra)*

Learning Priority 1: Develop understanding of numbers by counting and working with spatial patterns, and use numbers to represent quantities

Learning Priority 2: Develop number relationships, including comparisons, relation of one and two more or one and two less, relation to five and ten, and part-whole concepts

Grades PK-2: Foundations of Operation Sense *(Number, Operations, Algebra)*

Learning Priority 3: Explore operations of addition, subtraction, multiplication, and division through solving story problems, and develop computational strategies based on number relationships for addition and subtraction

Learning Priority 4: Develop foundational ideas of algebraic reasoning through solving problems in context and demonstrate initial understandings of equality and properties and relationships of operations

PK-12 Standard: Concepts and Connections in Geometry and Measurement

Grades PK-2: Foundations of Shape, Size, and Spatial Sense *(Geometry, Measurement, Number)*

Learning Priority 5: Develop spatial visualization and reasoning to interpret and describe the physical world with geometric ideas.

Learning Priority 6: Recognize measurable attributes of everyday objects, understand unit concepts and the process of measurement, and develop strategies to estimate and measure the size of objects

PK-12 Standard: Concepts and Connections in Data Analysis and Probability

Grades PK-2: Exploring Data Contexts *(Data Analysis, Number)*

Learning Priority 7: Explore questions about everyday experiences that can be answered with data, collect and organize data, and analyze the quantities as they relate to the context of the questions

Grades PK-2: Foundations of Number Sense (*Number, Algebra*)

Children grow tremendously in their knowledge of number and operations from pre-kindergarten through grade 2. They begin to understand the meaning of whole numbers as they learn to quantify their environment, talk about counting, explore spatial patterns, and represent quantities using objects, drawings and written numerals. One of the most important accomplishments during these early years is the development of children's number sense—their ability to use number relationships flexibly, meaningfully, and with confidence.

For young children, a first step in developing number sense is to learn the correct number word sequence. Then, they use the sequence and one-to-one correspondence to count a set of objects. Their knowledge of numbers reaches a new level when they understand that the last word in a counting sequence names the quantity of the whole set.

Young children have the ability to recognize the quantity of very small sets. Young children use that ability to quickly "see" or recognize how many items are in a set without counting by ones. They become fluent at recognizing familiar spatial arrangements like those on dice and dominos. They also learn to determine quantities in irregular and unfamiliar patterns by seeing smaller groups that make up the larger set. For example, a child says, "There are 7 dots because I saw a row of 4 dots and a row of 3 dots." This ability to see quantities as composed of smaller parts lays the foundation for both additive and multiplicative number relationships.

Once students have learned to connect number words with quantity, through counting by ones and seeing sets, they are ready to begin using numerals to represent quantities. With experience, children develop the flexibility to move back and forth among verbal, spatial, and written representations of quantities. This allows them to see that numbers are related to each other through a variety of number relationships.

Children need to develop several different types of number relationships during this grade band. One of the first relationships that many children notice involves comparison—learning to relate one number or quantity to another. It begins with a simple comparison between two quantities that answers the questions, "Which is more?" and "Which is less?" Building on this relationship, children can begin to see relationships of one and two more and one and two less, for example, 2 more than 2 is 4 and 1 less than 7 is 6. Another important relationship to develop is determining how numbers are related to the important benchmarks of 5 and 10, for example, children can use ten frames to see that 8 is three more than 5 and that 8 is two less than 10.

Part-part-whole relationships are another important aspect of number knowledge that children develop during this grade band. This relationship involves thinking about quantities as composed of two or more parts. For example, 10 can be thought of as 7 and 3, 5 and 5, 8 and 2.

Children's early work with the number word sequence, counting with one-to-one correspondence, and number relationships helps students in their first experiences with story problems and computation. Their work with story problems and computation in turn strengthens their number sense. As children move into the intermediate grades, they extend what they have learned about numbers and number relationships to develop ideas about place value, the structure of our base-ten numeration system, and computation with single- and multi-digit numbers.

Grades PK-2 Learning Continuum

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 1: Develop understanding of numbers by counting and working with spatial patterns, and use numbers to represent quantities	Number word sequence	Develop an increasing ability to count in sequence to ten and beyond	Develop an increasing ability to count forward and backwards by ones from any number within 1 to 100 Describe patterns in the number word sequence (e.g., 1, 2, 3, is repeated in 21, 22, 23 and 10, 20, 30)		Count forward from any number from one to 1000 by ones, tens, or hundreds
	Count and name quantities	Use one-to-one correspondence to count a small set of objects Understand that the last counting word tells "how many" objects are in a set	Use one-to-one correspondence to count a set of objects	Understand that counting a set of objects by ones or by groups of tens and ones results in the same amount Recognize that 10 can be thought of as one group of ten or ten singles and that 100 can be thought of as one group of 100 or ten groups of 10 or 100 singles	
	Recognize spatial patterns and name quantities	Represent collections with finger patterns for quantities up to five Recognize groups of one to five in common number cubes/domino arrangements	Name the quantity of objects in a spatial pattern, and discuss the ways the quantity is composed of smaller parts (e.g., spatial patterns include finger patterns, dominoes, ten frames, bead strings, and so on)		Name the quantity of dots or objects in an array and discuss the ways the array is composed of smaller groups
Learning Priority	Focus Areas	PK	K	1	2

Learning Priority 1: Develop understanding of numbers by counting and working with spatial patterns, and use numbers to represent quantities	Represent quantities	Represent quantities with objects and drawings and connect the quantity to its oral name	Use objects, drawings, and numerals to represent quantities to 100	Represent quantities to 1000 (e.g., numerals, drawings, objects) Explain the meaning of the digits in a multi-digit number. For example “25” represents a whole quantity, the “5” means five ones, the “2” means two tens or twenty ones, and 25 is equal to the sum of 5 and 20

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 2: Develop number relationships, including comparisons, relation of one and two more or one and two less, relation to five and ten, and part-whole concepts	Comparison of quantities	Visually identify whether two collections have the same amount or which has more	Compare quantities to determine which have the same amount, which have more objects, and which have fewer objects	Understand and use relationship words to compare quantities (e.g., equal, greater, smallest, more than, fewer than)	
	Relationships to other numbers		Visualize and describe relationships for numbers up to 20, including one and two more or less (e.g., 5 is one more than 4 and two less than 7) and relationships to 5 and 10 (e.g., 7 is 2 more than 5, 8 is two less than 10)		Identify relationships for numbers to 100 (e.g., 87 is 7 more than 80, three away from 90, and 13 away from 100)
	Part-part-whole concepts		Decompose collections of up to ten objects into two parts and make pictorial representations of the parts. (e.g., 6 is shown as 3 and 3, and as 1 and 5)	Decompose numbers to 20 by generating a variety of equivalent representations (e.g., 14 is 7+7 and 6+8 and 5+5+1+1+1+1)	Decompose numbers to 100 by generating a variety of equivalent representations

Grades PreK-2: Foundations of Operation Sense (*Number, Operations, Algebra*)

Solving story problems is a large part of children's mathematical work in pre-kindergarten through grade 2. This work allows children to build on their intuition and emerging counting skills to solve problems, and leads toward an understanding of operations. It also provides a context for developing flexible and meaningful strategies for computation.

Over time children develop increasingly sophisticated strategies for solving story and number problems involving single-digits. Initially, they use physical objects, fingers, and drawings to model and solve problems involving a variety of problem situations, including joining, separating, part-part-whole, comparison, grouping, and partitioning situations. Children transition from modeling all quantities and actions in a problem to modeling only some of the quantities in a problem, and using counting strategies such as counting on and counting back. Eventually, children solve story problems using strategies based on number relationships. Once children are able to solve problems involving single-digit computations using number relationships, they are ready to start building fluency. Fluency involves using strategies that take children beyond counting to meaningful, flexible, and efficient use of number relationships. For example, to add 8 + 5 using a relationship to ten, children first figure out how to break apart the 5 into quantities of 2 and 3 so that they can add 2 + 8 to get ten and then add on the 3 to get 13. To subtract 12 – 7, children could find the difference by building up through ten and reasoning that 7 plus 3 builds to ten and then 2 more builds to 12 so the answer must be 3 and 2 or 5.

The strategies and number relationships that children develop as they work with single-digit computation lay the groundwork for multi-digit computation. Children's development of strategies for solving addition and subtraction problems with larger numbers parallels their earlier work with single-digit numbers. They often begin by using base-ten materials to model the quantities and actions in the problem. Many children then proceed to use counting strategies, counting on or back by hundreds, tens, and ones. Ultimately, students develop more abstract strategies, such as incrementing, compensating, and tens and ones strategies, that build on the rich web of number relationships they are constructing. At this grade band, children should also use objects to explore and model situations with equal groups to build a foundation for multiplication and division.

Beginning in the early grades, children use what they have learned through solving story and number problems as they learn how to express their mathematical ideas using words, numbers and conventional symbols. This allows them to discuss equations, notice relationships and make conjectures about the arithmetic they have been learning (e.g., it's okay to start with the larger number when you add, because order doesn't matter). Through their discussions about equations, children begin to develop meaning for the equal sign—it means "has the same value" not "the answer comes next."

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 1: Explore operations of addition, subtraction, multiplication, and division through solving story problems, and develop computational strategies based on number relationships for addition and subtraction	Solve story problems and	Investigate simple joining, separating, grouping, and partitioning situations in a variety of ways (including modeling or "role playing" the situations and representing them using objects or drawings)		Represent, solve, and discuss problems in context including a variety of situations (e.g., joining, separating, part-part-whole, comparing, grouping, measuring, and partitioning)	
	Strategies for single-digit computation	Find solutions to story problems by using physical objects to directly model the actions or relationships among the quantities in story problems		Find solutions to problems by modeling with objects or drawings, and by using counting strategies (e.g., counting on, counting back, skip counting) Develop strategies to find sums of single-digit numbers and differences of related subtraction problems by using number relationships and derived facts (e.g., use a relationship to ten to solve 8 + 6, (8 + 2) + 4 = 10 + 4 = 14; solve 13 – 6 with a double, (6 + 6) + 1 = 13, so the difference is 6 + 1 or 7) Fluently use efficient strategies to solve single-digit addition problems, and begin to develop fluency for the related subtraction problems	
	Strategies for multi-digit computation			Solve problems with multi-digit numbers by directly modeling the actions or relationships among the quantities or by using counting strategies	

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 1: Explore operations of addition, subtraction, multiplication, and division through solving story problems, and develop computational strategies based on number relationships for addition and subtraction				Develop strategies using mathematical tools to add multi-digit numbers (e.g., cubes, base-ten blocks, hundreds chart, or open number line)	Develop and use strategies based on number relationships to add and subtract multi-digit numbers (e.g., incrementing, 38 + 24 = 38 + 2 + 22; compensating, 38 + 24 = 40 + 24 – 2; and tens and ones, 38 + 24 = 30 + 20 + 8 + 4)

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 2: Develop foundational ideas of algebraic reasoning through solving problems in context and demonstrate initial understandings of equality and properties and relationships of operations	Meaning of the equal sign			Begin to understand that the equal sign means “has the same value” rather than “the answer comes next” by investigating a variety of equations (e.g., 10 = 4 + 6, 5 + 5 = 4 + 6, 8 = 8)	
	Properties and relationships of operations			Develop initial ideas about the properties of operations as they engage in problem solving and discussions about equations (e.g., “The order doesn’t matter when you add” or “If you add 0 to a number, you get the number you started with.”) Develop initial ideas about the inverse relationship between addition and subtraction	

Grades PreK-2: Foundations of Shape, Size, and Spatial Sense (*Geometry, Measurement, Number*)

Children in prekindergarten through grade 2 develop spatial sense as they examine, visualize, and communicate about the shapes of objects and about differences in position, size, shape, and the relationships between objects. Young children begin with investigations of three-dimensional objects as they start interpreting the physical world with geometric ideas and make connections to two-dimensional shapes. They find shapes in their environments and describe them in their own words. This informal language and experiences form the basis for allowing geometric language to emerge in meaningful contexts. Children then learn to identify, name, and describe a variety of two-dimensional and three-dimensional shapes, presented in a variety of ways (e.g., with different sizes or orientations). They draw, construct, compose, and decompose shapes to build an understanding of part-whole relationships of regions and shape, as well as the properties of the original and composite shapes. As they construct two-dimensional and three-dimensional models of shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, in the process developing a background for measurement and initial understandings of such properties as congruence and symmetry.

Children strengthen their spatial sense as they directly and visually compare the "size" of objects. Their study of size begins by identifying measurable attributes such as length, capacity, and mass and solve problems by making direct comparison of two objects on the basis of those attributes. They consider these "size" qualities of objects as they identify objects as "the same" or "different," and then "more" or "less." Direct measurement of length consists of comparing objects by lining up items side by side, beginning at a common base; indirect measurement of length occurs when it is not possible to physically line up the objects. Comparing capacity directly involves filling one container and then pouring the contents into another to find which holds more and which holds less; indirect measurement of capacity involves comparing the capacities of two containers to that of a third referent container. Comparison of mass directly involves placing two objects on a balance scale simultaneously and comparing the mass of one with that of the other. Direct measurement of area involves placing one surface on top of another to see which "sticks out."

Children strengthen their sense of number as they solve problems to determine "how much." These early activities build the foundation for establishing an understanding of unit concepts and the measurement process and provide a meaningful context for counting and for connecting the ideas of number and number sense to the real world. The study of the measurement process begins with non-standard units and then moves to standard or customary units after children have a deeper understanding of attributes and the process of measuring a particular attribute. Children come to understand measures as an iteration of units and use rulers and other measurement tools with that understanding. They can explain the inverse relationship between the size of a unit and the number of units used in a particular measurement. Children reflect on and communicate the need for standard units of measure and begin using standard metric and customary units, particularly for length. Meaningful measurement and estimation of measurements depend on a personal familiarity with the unit of measure being used, thus it is essential that children develop their own personal measurement benchmarks for standard units. For example, a child might know that the distance from the floor to her knee is one foot or the width of a finger is one centimeter. These personal benchmarks increase children's familiarity with units, prevent errors in measurement, and aid in the meaningful use of measurements.

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 1: Develop spatial visualization and reasoning to interpret and describe the physical world with geometric ideas	Compose, decompose, and describe shapes	Build and describe three-dimensional objects and structures using a variety of blocks and materials	Sort and classify three-dimensional objects and identify similarities and differences based on their geometric attributes	Investigate and predict the results of putting together and taking apart two-dimensional and three-dimensional shapes Describe attributes and parts of two-dimensional and three-dimensional shapes Recognize, name, build, draw, compare, and sort two-dimensional and three-dimensional shapes	
			Create designs, pictures, and structures by combining two- or three-dimensional shapes and describe how the shapes are used and relate to each other	Investigate questions about classes of shapes, such as, "Which of these shapes are triangles? How are triangles different from squares? What makes this shape a cube?"	
	Spatial relationships	Demonstrate spatial rotation of pieces when putting together puzzles	Identify two-dimensional shapes in many different sizes and orientations (e.g., triangles that are fat or skinny, tall or short, point to the side or point down)	Recognize polygons in rotated positions and talk about how they are still the same shape (e.g., it's still a square just turned)	
		Demonstrate visual matching of shapes and designs that are the same size and shape		Explore and draw triangles and other polygons in many different sizes and orientations and use informal language to describe how they are the same and different	

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 1: Develop spatial visualization and reasoning to interpret and describe the physical world with geometric ideas	Spatial relationships		Build designs that have line or rotational symmetry and describe the regularity in the design	Recognize and apply slides, flips, and turns and describe the effects of these movements on shapes	
			Identify and describe lines of symmetry in everyday objects		Recognize and create shapes that have line and rotational symmetry
			Identify and describe how two-dimensional representations relate to three-dimensional structures and objects		

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 2: Recognize measurable attributes of everyday objects, understand unit concepts and the process of measurement, and develop strategies to estimate and measure the size of objects	Measurable attributes	Recognize and describe measurable attributes of objects such as length, mass, or capacity		Select and justify appropriate units for quantifying specific attributes of objects	Identify standard units of measurement to quantify the size of objects
		Compare measurable attributes of objects, including comparison of gross differences of two objects using language such as bigger, longer, taller, or heavier		Develop and discuss strategies to quantify measurable attributes of objects	Establish and use personal referents for commonly used measures to develop a sense of the relative size of standard units
		Develop and describe strategies to order two or three objects by comparing them directly (e.g., side by side) or indirectly (compare both with a third object)		Use non-standard units to estimate and measure attributes of objects	Estimate and use appropriate tools to determine measurements in everyday problem situations

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority 2: Recognize measurable attributes of everyday objects, understand unit concepts and the process of measurement, and develop strategies to estimate and measure the size of objects	Unit concepts		Explore use of non-standard units (e.g., children's bodies, footsteps, or strides) to quantify distances and other measurable attributes of objects	Measure objects through iteration of a unit by using and counting multiple copies of the unit	Communicate and demonstrate the need for equal-sized units and standard units
					Know that any point on a measurement tool can act as the starting point of a measurement (e.g., in using a broken ruler)
				Explain the importance of stating the quantity with the name of the unit in reporting measurements	Describe how the same measurement can be communicated using different unit names
				Describe what happens when something is measured with small units versus larger units	Recognize how subdividing or using smaller units will increase the precision of a measurement

Grades PreK-2: Exploring Data Contexts *(Data Analysis, Number)*

Children in prekindergarten through grade 2 strengthen their number sense and consider ways to quantify, compare, and talk about their world through the exploration of data. This study of comparing quantities within the context of children's everyday experiences is what distinguishes data analysis from just working with number. It is the context that provides meaning for the numbers as children wonder about aspects of their world and consider the interests of their classmates in comparison to themselves. As children examine data it should be to answer questions about themselves and their environment and responses should always be related back to the original question and context. For example, to say "five is the most" is not sufficient. We want children to say that five people chose pink as their favorite color.

Children gather data to answer questions about themselves and their surroundings. The questions might be posed by the teacher, but children also need to gain experiences and skill in asking questions that can be answered by collecting data. Younger children might be asked to pose a question that can be answered with two choices, such as "yes" or "no," and then collect data from their classmates. By the end of this grade band, students should be posing questions for collecting data that have multiple response categories or that require a numeric response.

Children initially work with data by sorting objects, including themselves, and quantifying the groups. For example, they might sort a collection of buttons based on color to find out how many they have of each color and then show and talk about their sorts. As children grow in their ability to ask questions and look for answers, they enjoy collecting data through informal surveys and observations. Children first display their data by showing all of the information they gathered without organizing or categorizing. For example, they list the names of each classmate and the food that each child likes. Then they consider ways to categorize the information they collect and display the data.

A very important aspect of data analysis is for students to consider their own ways to organize and represent data. The goal is for children to bring meaning to the data they have collected, so they need experiences to muddle around with the data and consider different ways to organize and create their own displays of the data so that they talk about what the data might be telling us. These informal and student-created data displays with objects or pictures form the foundation for understanding the components of more standard displays for communicating information to others. By the end of second grade, students should have some familiarity with organizing and representing data with charts, tables, simple bar graphs, and Venn diagrams, as well as posing and answering questions about information in these data displays. As students describe and analyze data, they are able to notice individual aspects of the data—where their own data are on the graph, for instance, or what value occurs most frequently in the data set, and are only beginning to move toward seeing a set of data as a whole in describing its shape and spread.

Although data explorations in the early grades require significant teacher support, these experiences provide a context for developing number ideas and build a foundation for further study of data analysis in grades 3-5 and beyond. Some valuable aspects of early explorations in data analysis are classroom conversations about how to gather, organize, and analyze information to answer questions that are of interest to students and to raise additional questions raise additional questions about the data and the context under investigation.

Learning Priority	Focus Areas	PK	K	1	2
Learning Priority: Explore questions about everyday experiences that can be answered with data, collect and organize data, and analyze the quantities as they relate to the context of the questions	Collect and organize data to answer questions	Explore "data questions" that arise in the context of classroom or practical situations (i.e., questions that can be answered by collecting data, such as, Does our class want indoor or outdoor recess today?)		Pose simple questions about everyday experiences that can be answered by collecting data	
		Sort and organize "real" objects (e.g., children, photos, hats) to create displays that allow quantities to be compared		Collect, organize, and represent data (numerical and categorical) so it is useful in answering questions and communicating information clearly to others	
	Analyze and compare data in relation to the context	Use displayed and organized data to make observations and compare quantities in relation to the problem context		Use data displays to make observations and note features of the data	
		Formulate an answer to the original question and state curiosities or make conjectures about the context		Pose and answer questions about the data, summarize an answer related to the original question, and raise additional questions, wonderings, and conjectures about the context	