

Revised WI Model Academic Standards for Mathematics – Grades 6-8

PK-12 Standard: Concepts and Connections in Number and Algebra

Grades 6-8: Numeric and Algebraic Reasoning (*Number, Algebra*)

Learning Priority 1: Represent and communicate with real numbers when solving mathematical, real world and non-routine problems.

Learning Priority 2: Compute fluently and make reasonable estimates using rational numbers.

Learning Priority 3: Understand and use ratios, proportions, and proportional reasoning.

Learning Priority 4: Understand patterns, relations, and functions and their connections within and outside of mathematics.

Learning Priority 5: Model and solve mathematical problems in a variety of meaningful contexts using algebraic expressions, equations, and inequalities.

PK-12 Standard: Concepts and Connections in Geometry and Measurement

Grades 6-8: Geometric Reasoning (*Geometry, Measurement, Algebra*)

Learning Priority 1: Communicate an understanding of geometric attributes and relationships.

Learning Priority 2: Use visualization, spatial reasoning, and geometric modeling to connect geometric representations to algebraic relationships

Learning Priority 3: Understand measurable attributes, processes, systems and units of measurement and use appropriate tools and techniques to work with both direct and indirect measurement.

PK-12 Standard: Concepts and Connections in Data Analysis and Probability

Grades 6-8: Quantitative Reasoning (*Data Analysis, Probability, and Algebra*)

Learning Priority 1: Formulate questions and design statistical investigations that can be answered by data collection and analysis.

Learning Priority 2: Collect and display data with a variety of representations, justifying the appropriateness of methods used.

Learning Priority 3: Analyze data from organized displays and numerical descriptors; interpret information, make predictions, and draw conclusions with supporting arguments.

Learning Priority 4: Use a variety of counting techniques, experiments, and simulations to determine probabilities of events.

Draft.... for discussion and comment

Middle Level Learners

The middle level grades are a time of transition. It is in grades 6-8 that students make the greatest growth changes, intellectually, physically, emotionally, and spiritually, beyond those of the first few years of life. Middle level mathematics follows suit. In the few short middle level years, the mathematics that students do moves from intuitive and experiential to technical and, perhaps, axiomatic. Middle level students become capable of handling information input and constructive analysis. From doing and image making and having, they move to noticing properties, to formalizing ideas, to observing and structuring, and, hopefully, to inventing. They welcome challenge and appreciate support. It is in grades 6-8 that students often solidify the way they perceive themselves as students of mathematics. Motivation, confidence-building, and enabling students to handle substantive mathematics are essential components of middle school mathematics programs.

The standards, then, for middle level mathematics need to be ambitious and rigorous. The mathematical tasks that are undertaken need to be engaging, memorable, challenging, and worthwhile and need to be delivered so that the energies of adolescents can be channeled toward investigating, conjecturing, hypothesis-testing, concluding, checking and revising, and deepening conceptual understanding. Ensuing self-confidence helps students build productive dispositions and willingness to tackle problem-solving. Opportunities need to be provided for students to continue to develop procedural fluency and to increase their strategic competence, knowing strategies to use and how to use them. Building on strong foundations, students can then hone adaptive reasoning skills. Deepening thinking skills is a goal of middle level mathematics; students need to be asked to recall, to convey understanding, to apply knowledge, to analyze situations, to synthesize and connect ideas, to evaluate conclusions, and, ultimately, to create.

The following grades 6-8 standards address the learning continuum inherent in the total standards package. The learning priorities reflect the major areas of concentration for the grade span. The focus areas identify the components of importance to total development of each of the priorities. Since many of the skills and concepts embedded in the focus areas are developed over the course of time, often a target statement extends through two or more grade levels. Certainly some targets are pinpointed specifically to a grade level. Teachers will welcome the specificity embedded in this set of standards for the direction they give.

The mathematics program for *all* students in the middle grades should address three model academic standards that connect previously identified content strands—concepts and connections in number and algebra, concepts and connections in geometry and measurement, and concepts and connections in data analysis and probability. Explorations of the content of the middle level curriculum should be situated in rich, memorable contexts and should incorporate the mathematical process skills—problem solving, conjecture, reasoning and proof, representation and visualization, communication and reflections, and connections.

The National Council of Teachers of Mathematics notes that mathematics needs to be “interesting and relevant to our students” (*Principles and Standards for School Mathematics*, p. 31). It is hoped that these revised WMAS Standards provide impetus for both.

Grades 6-8: Numeric and Algebraic Reasoning (*Number, Algebra*)

Rational number understanding, proportionality, and algebraic reasoning are major components of middle school mathematics.

In grades 6-8, students continue to build understanding of numbers, number representations and relationships, systems of numbers, meanings of operations and their relationships, and they extend work with fluency and estimation. They increasingly become aware of the capabilities of technologies and of the appropriate selection of and use of each. Mathematics in grades 6-8 bridges the arithmetic, number sense, and numerical fluency developed in the elementary grades with the algebraic thrust of the high school. Rich, ambitious, and varied problems that address quantitative real-life situations prepare students to meet the demands of the world they encounter in the middle school and move them toward the more formal high school investigations that ensue.

The middle level curricula should offer opportunities for building deep understanding of rational number, rational-number computation and estimation, and fluency and flexibility in the use of rational numbers—fractions, decimals, percents, and integers—in solving problems. Rational number explorations should involve a range of real-world topics such as fair sharing, taxes, tips, discounts, etc. The need for exciting, useful, and creative contexts calls for the use of several models—linear, area, grid, partition—to facilitate development of rational number understanding.

Through problem solving and reasoning and establishment of connections within mathematics and with other domains, understanding of proportionality is established. Facility with percent, similarity, scaling, linear equations, slope, relative-frequency histograms, and probability develops and augments concepts of proportionality. A focus on meaningful applications of proportional reasoning capitalizes on numeric thinking and moves students to analytic reasoning.

Algebraic reasoning is a major component of middle level mathematics. Students recognize linear and non-linear relationships and work effectively with these relationships in various representations—tables, graphs, and equations—in solving problems set in meaningful contexts. Students develop facility with reasoning, modeling, analyzing, solving, and contemplating the reasonableness of results in algebraic situations, manipulations, and problem solving. Students recognize the beauty of succinct and efficient algebraic reasoning in problem solving and use it as a tool for addressing substantive, engaging problems.

These students, over the course of the curricular sequence, move through a progression of numerical understanding that moves from the set of whole numbers to integers to rational and irrational numbers, the set of real numbers. In developing such understanding, students need to engage in thoughtful activity that allows them to consider structure, conjecturing and verifying, thinking hypothetically, comprehending cause and effect, and abstracting and generalizing.

Middle level, students develop useful methods to compute with rational numbers, proportional reasoning, and algebraic modeling that “make sense.” They develop their own methods and learn to appreciate traditional algorithms, comparing and contrasting the two. Additionally, they develop and adapt procedures for mental computation and estimation and flexibility in thinking with real numbers.

Though the span of grades 6-8 is small, the extension of numeric understanding and the development of algebraic reasoning during those years is vast and imperative and moves the student toward substantive mathematics.

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 1: Represent and communicate with real numbers when solving mathematical, real world, and non-routine problems</p>	<p>Number Theory</p>	<p>Distinguish between prime and composite numbers and present a convincing argument that a number is prime</p> <p>Understand and determine greatest common factors, least common multiples, and relatively prime numbers</p> <p>Apply divisibility tests to numbers</p>		<p>Use prime factorization (using exponents) to determine GCF, LCM, and relatively prime numbers</p> <p>Distinguish between rational and irrational numbers</p> <p>Extend the relationships of factors, multiples and rules of divisibility to an algebraic setting. (e.g. recognize that an algebraic term is made up of factors and identify algebraic expressions with common factors)</p>
	<p>Modeling</p>	<p>Represent fractions, decimals, and percents using physical and visual models (e.g., concrete objects, drawings, fraction strips, number lines, fraction wheels, area models, grid models, partition models)</p>	<p>Represent integers using physical and visual models (e.g. number lines and two-color chips)</p>	<p>Represent irrational numbers as lengths using physical and visual models (e.g. geo-boards, dot or grid paper.)</p>

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 1: Represent and communicate with real numbers when solving mathematical, real world, and non-routine problems</p>	<p>Order</p>	<p>Compare and order positive rational numbers with symbols ($>$, $<$, \leq, \geq, \neq, \approx); represent their location on a number line</p> <p>Understand and apply the conventions for algebraic order of operations</p> <p>Describe what it means for a number to be between two numbers</p>	<p>Use understanding of integers and negative rational numbers to locate and order these numbers on a number line</p>	<p>Compare and order irrational numbers and represent their approximate location on a number line</p>
	<p>Properties</p>	<p>Recognize and use the properties of operations, including associative, commutative, distributive, and identity, and identify which properties are true for which operations</p> <p>Describe the effect of multiplying or dividing a number by one, by a number between zero and one, and by a number greater than one</p> <p>Understand that division by zero is undefined</p>	<p>Understand absolute value</p> <p>Apply distributive and other properties to integer and rational expressions in problem solving situations</p>	
	<p>Order of Operations</p>	<p>Understand and apply the conventions for algebraic order of operations to equations and expressions without and with variables</p>	<p>Extend and apply the conventions for algebraic order of operations to algebraic expressions</p>	
	<p>The Concept of Inverse Relationships</p>	<p>Recognize and apply the inverse relationships of addition and subtraction, multiplication and division</p> <p>Understand and use the opposite and reciprocal of a rational number</p>	<p>Extend understanding of inverse relationships, opposites, and reciprocals to integer and rational number operations</p>	<p>Explore, explain and apply the inverse relationship between squares and square roots</p> <p>Recognize common right triangle triples (e.g., 3, 4, 5) in the context of the Pythagorean Theorem and its converse</p>

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 2: Compute fluently and make reasonable estimates using rational numbers</p>	<p>Equivalence</p>	<p>Develop and apply procedures for generating equivalent fractions and use these equivalent forms in estimation, computation, and ordering situations</p> <p>Generate equivalences among fractions, decimals, and percents and translate among representations both with and without context</p>	<p>Generate equivalent ratios by scaling up and scaling down</p>	<p>Develop and apply a variety of strategies, including the use of scale factors, to solve problems involving similarity and proportionality</p>
	<p>Computational Strategies, including estimation</p>	<p>Select appropriate methods and tools for performing computations in problem solving including mental computation, estimation, paper and pencil, and technology</p> <p>Estimate the solutions and then solve problems in contexts involving fractions, decimals, and percents and verify the reasonableness of results</p> <p>Develop an understanding of and fluency with algorithms for computing with non-negative rational numbers</p>	<p>Estimate solutions then solve problems in contexts involving integers and rational numbers; verify solutions and compare estimation techniques</p> <p>Develop understanding of and fluency with algorithms for computing with integers and positive and negative rational numbers</p>	<p>Expand percent applications to complex problems (e.g. compound interest), using results to develop financial literacy and decision-making skills</p>

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
Learning Priority 3: Understand and use ratios, proportions, and proportional reasoning	Part-whole and part-part relationships	Use part-whole and part-part relationships with clarity about “the whole” in a variety of contexts to explore rational numbers and develop proportionality concepts, including rate and ratio		
	Proportional relationships	Distinguish differences among proportional and non-proportional relationships		
	Percent	Solve a wide variety of percent problems, including: discounts, interest, percent increase or decrease, taxes, tips and similarity		
	Representations	Graph proportional relationships and identify the unit rate as the slope		
	Scale Factors	Use scale factors in numeric, algebraic, and geometric contexts		

Learning Priority 3: Understand and use ratios, proportions, and proportional reasoning	Equivalence	Develop and apply procedures for generation of equivalent positive fractions, and use these equivalent forms in estimation, computation, and ordering situations	Extend equivalency to include all rational numbers
			Extend and develop equivalency concepts to include algebraic terms and variables

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
Learning Priority 4: Understand patterns, relations, and functions, and their connections within and outside of mathematics	Change	Generate a rule to describe the change in a numeric or geometric pattern and be able to justify its selection	Identify and describe linear change as represented in tables, graphs, and equations	Explore, analyze, and represent patterns of linear and non-linear change (e.g., direct and inverse variations, exponential growth and decay)
	Patterns and Generalizations		Describe, recognize, and interpret representations of patterns in relevant contexts, including patterns on coordinate grids Analyze and generalize linear and non-linear patterns in a variety of forms (e.g. tables, graphs, and equations)	
	Linear and Non-linear Relationships	Represent relationships between quantities using oral and written descriptions, tables, graphs and student-generated formulas	Represent a linear function in context with oral and written descriptions, table, graph, or symbolic expression, make connections among these representations and communicate the strengths and weaknesses of each representation	Identify functions as linear or non-linear and compare and contrast their properties when the function is presented in table, graph or equation form and flexibly translate among these representations
	Interpreting Graphs	Match a realistic story to a representation on a given graph	Create a realistic story that a given graph might represent	Generate a variety of realistic stories to match graphical representations with various shapes

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 5: Model and solve mathematical problems in a variety of meaningful contexts using algebraic expressions, equations, and inequalities</p>	<p>Relationships within and between algebraic expressions, equations, and functions</p>	<p>Distinguish among expressions, equations, and functions and communicate the differences</p> <p>Evaluate, interpret, and construct simple algebraic expressions</p> <p>In a given context recognize and explain the meaning of each term in an algebraic expression or equation</p>	<p>Evaluate, interpret, and construct simple algebraic expressions</p>	<p>Create an expression, equation, or function to appropriately model a contextual situation including nonlinear relationships</p> <p>Identify and transform expressions into equivalent expressions and determine whether two algebraic expressions are equivalent</p>
	<p>Variables and Formulas</p>	<p>Recognize and interpret the role of variables</p> <p>Understand that when a variable is used in an expression or equation, it represents multiple values (e.g. $n \times \\$7 =$ admission price for n people)</p> <p>Evaluate formulas in a context.</p>	<p>Model a real-world situation by creating a corresponding algebraic expression or equation using one or more variables</p> <p>Solve for a specific variable in terms of the others in formulas or equations involving several variables with and without context</p>	<p>Describe how a change in one variable affects another variable in linear and non-linear equations; graphically and symbolically model the contextual situation</p> <p>Create and justify formulas (including on spreadsheets) in contexts such as measurement, finance, and data analysis</p>

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 5: Model and solve mathematical problems in a variety of meaningful contexts using algebraic expressions, equations, and inequalities</p>	<p>Solving equations and inequalities</p>	<p>Solve linear equations and evaluate expressions through numerical substitution</p>	<p>Using numerical substitution and/or algebraic procedures solve single-variable linear equations and inequalities (using =, >, <, ≤, ≥)</p> <p>Graph the solution on a number line and interpret the solution(s) in terms of the context of the problem</p>	<p>Predict the number of solutions for an equation and recognize that solving an equation means finding all of its solutions</p> <p>Solve linear inequalities and equations algebraically and interpret the solution(s) in terms of the situation</p>
	<p>Coordinate representations</p>	<p>Graph linear functions</p>	<p>Determine the slope (including slope triangles) and y-intercept of a linear function; evaluate the slope intercept form of a linear equation ($y = mx + b$) for a variety of x and y values</p>	<p>Solve systems of linear equations on a coordinate grid (geometrically)</p>

Grades 6-8 Focus Areas and Learning Continuum

Grades 6-8: Geometric Reasoning (*Geometry, Measurement, Algebra*)

Geometric reasoning in the middle level connects the concepts of geometry, measurement, and algebraic reasoning and uses them in problem solving, analysis of mathematical situations, and in mathematical argument and discourse. Students transition from concrete (hands-on) or visual representations to internalized abstract representations. They learn to represent algebraic ideas with geometric representations and geometric ideas with algebraic renderings.

Utilizing informal knowledge of geometric concepts such as points, lines, planes, plane figures, and space figures, middle school students explore attributes of geometric two- and three-dimensional figures. They draw figures, measure, visualize, compare, transform, and classify geometric objects, and identify relationships among angles and side lengths, perimeters and areas, and volumes. Using the knowledge gained from manipulation and visualization, students use inductive and deductive reasoning to make conjectures, to validate them, and to develop plausible arguments regarding relationships.

Students learn to use coordinate geometry to specify locations, to describe spatial relationships, and to identify properties of figures such as regular polygons and figures with parallel and perpendicular sides. Algebraic techniques are tools for reasoning about geometric shapes on a coordinate grid. Geometric images are useful aids to algebraic reasoning about linear equations and inequalities. Analyzing properties of equations helps in identifying geometric relationships that characterize various familiar geometric shapes.

Measurement connects number and geometry. In the middle grades, students build on previous informal measurement experiences with length, area, and volume, measurement units and measurement systems. Students begin to develop deeper understanding of selection of appropriate units, precision, conversion of units, scaling, etc., and use this knowledge to solve problems involving area and perimeter and volume. Students also work with angle measurement and, using ratio and proportion, solve problems using scale, similarity, and indirect measure.

Manipulation of geometric figures through transformations, use of symmetry and congruence, and awareness of orientation, helps students investigate geometric objects and analyze mathematical situations. Use of dynamic software augments manipulative understanding; being able to virtually reflect, rotate, dilate, and move two- and three-dimensional objects, helps students build visual images useful for geometric problem solving.

Geometric modeling allows students to visualize geometric situations and contributes to building bases for spatial reasoning. Being able to visualize and to draw and construct geometric two- and three-dimensional figures is basic to being able to solve geometric problems. Students need to be able to compose and decompose geometric figures. Use of tools such as networks facilitates the solving of efficiency problems. Students in the middle school need to develop understanding of the Pythagorean relationships as they connect geometry and algebra. Identity relationships can also be well illustrated using geometric tools.

Geometry, measurement, and algebraic reasoning lend themselves well to curricular connections. Nature, art, and science have geometric connections and patterns. Knowledge of the golden ratio, the rigidity of the triangle, and the usefulness of the Pythagorean relationship in indirect measure are evidences of applied geometry, "earth measure."

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 1: Communicate an understanding of geometric attributes and relationships</p>	<p>Attributes of two and three dimensional shapes</p>	<p>Describe with precision and classify two and three dimensional figures by their attributes (e.g. angles, sides, perimeter, area, and volume)</p> <p>Recognize, understand, and communicate the relationships among and between two-dimensional and three-dimensional figures (e.g. the base of a particular cone is a circle)</p>		
	<p>Geometric Reasoning</p>	<p>Make conjectures and investigate relationships when given constraints (e.g., tiling, fixed perimeter, fixed area)</p>	<p>Prove or invalidate conjectures concerning geometric ideas</p> <p>Analyze geometric reasoning paths</p>	<p>Create arguments (inductive and deductive) concerning geometric ideas and relationships</p> <p>Critique paths of geometric reasoning, including conjectures, deductions and conclusions</p>
	<p>Similarity</p>	<p>Develop an understanding of similarity in geometric figures, including identifying similar figures and comparing corresponding parts using scale factors and ratios</p> <p>Describe relationships among corresponding sides of similar figures and construct similar polygons</p>		
	<p>Transformations</p>	<p>Describe sizes, positions and orientations of shapes</p> <p>Examine and describe congruence, similarity, reflections, and line or rotational symmetry of figures using transformations and formal terminology</p>		<p>Use coordinate grids to transform figures and to predict the effect of transformations, including dilations, translations, rotations, and reflections</p>

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 2: Use visualization, spatial reasoning, and geometric modeling to connect geometric representations to algebraic relationships</p>	<p>Visualization and Representations</p>	<p>Use visual tools and manipulatives to model problems (e.g. reflective devices, isometric drawings, mat plans, paper-folding, nets, geo-boards, ominoes, tiles, cubes)</p> <p>Solve problems involving visualizations such as sketching a third view of a three-dimensional object when given the side view and the top view</p>	<p>Visualize and construct three-dimensional objects using two-dimensional representations</p> <p>Draw and/or use two-dimensional representations of three-dimensional objects to visualize and solve problems (e.g. surface area and volume)</p> <p>Analyze how perimeter, side length, height, area, and volume change under scaling of one or more dimensions of a figure</p>	<p>Relate a geometric representation to an algebraic expression, and take a algebraic expression and represent it geometrically (on a coordinate grid)</p>
	<p>Side relationships in polygons</p>	<p>Explore side and angle relationships in polygons (e.g., the sum of the lengths of two sides of a triangle is longer than the length of the third side)</p>	<p>Draw geometric objects with specified properties such as side lengths and angle measures</p>	<p>Investigate, justify and use the Pythagorean Theorem to find distances</p> <p>Use the Pythagorean Theorem to construct the square roots of positive integers</p>
	<p>Dynamic Explorations</p>	<p>Using dynamic explorations (e.g., computer software) develop conjectures regarding geometric relationships and communicate mathematical arguments</p>		

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 2: Use visualization, spatial reasoning, and geometric modeling to connect geometric representations to algebraic relationships</p>	<p>Coordinate Geometry</p>	<p>Identify, locate and plot points on coordinate grids</p>	<p>Specify locations and describe spatial relationships using coordinate geometry in all four quadrants</p>	<p>Understand and apply the similarity of slope triangles in algebraic and geometric settings</p> <p>Use coordinate geometry to examine lines and special geometric shapes (e.g., regular polygons, or those with parallel and perpendicular sides)</p>

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 3: Understand measurable attributes, processes, systems and units of measurement and use appropriate tools and techniques to work with both direct and indirect measurement</p>	<p>Measurement</p>	<p>Identify a system (metric or customary) and appropriate units, tools, techniques, benchmarking and estimation strategies to measure a given attribute with identified accuracy (for length, weight/mass, liquid capacity, temperature, and elapsed time)</p> <p>Solve real world problems which require conversion within systems (e.g., meters to centimeters, or feet to inches)</p> <p>Make approximate comparisons between metric and customary units (e.g., a liter and a quart are about the same; a kilometer is about six-tenths of a mile) to solve contextual problems</p>	<p>Apply similarity concepts to determine indirect measures, including measures that could not otherwise be determined</p>	<p>Determine measurements indirectly using estimation, ratio and proportion, geometric formulas, the Pythagorean Theorem, and geometric relationships</p>
	<p>Area and Volume</p>	<p>Explore, understand, and use strategies to determine perimeter and area of regular and irregular figures</p> <p>Use measurement to solve problems in relevant contexts, including area, perimeter, and scale drawings</p> <p>Use measurement to solve problems in relevant contexts involving proportional reasoning, surface area, and volume</p>		
	<p>Angle measurement</p>	<p>Identify, measure, and draw angles (to the nearest degree) and recognize benchmark angles</p> <p>Explore and determine angle measures of polygons, including the sum of the measures of the interior angles of a triangle equals 180°</p>	<p>Relate angles to benchmark angles and identify pairs of angles as complementary, supplementary, adjacent, or vertical</p>	<p>Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals</p> <p>Utilize knowledge of angle measures of polygons (interior, exterior, central) to solve problems in context</p>

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 3: Understand measurable attributes, processes, systems and units of measurement and use appropriate tools and techniques to work with both direct and indirect measurement</p>	<p>Precision and accuracy</p>	<p>Understand that all measurements contain error</p>	<p>Select and justify an appropriate level of precision with which to measure</p>	<p>Recognize the role that the degree of accuracy with which recorded measures have been made has in interpreting the measurement context (e.g., number of miles to the sun)</p> <p>Determine the appropriate level of precision with which to report results in a given measurement context</p>

Grades 6-8: Quantitative Reasoning (*Data Analysis, Probability*)

Quantitative reasoning at the middle level bridges numeric and algebraic reasoning and geometric reasoning and measurement. Students relate to data-driven contexts meaningful to them in formulating questions that can be addressed by collecting data, in collecting and organizing the resulting data, in displaying the data, in analyzing their findings and in developing inferences and predictions based on data. Additionally, working with described data, students in grades 6-8 understand and apply the basic concepts of probability, the likelihood of events, the odds of a happening, and combinations and permutations. In all explorations, middle school students work to develop problem solving skills and strategies and learn to evaluate the reasonableness of conclusions and the thought processes involved in hypothesis and proof. They learn to use multiple representations to gain access to problems and to communicate results. They work in integrated settings and consider the connections with the mathematics, with other disciplines, and with problems encountered in daily living.

A major emphasis in dealing with data in the middle school is decision-making regarding appropriate data techniques. Students learn to identify questions that can be well-answered by data collection and analysis. They learn to compare characteristics within and between populations. They also learn to find data sources for analysis. Once data has been obtained, selection of appropriate display methods is considered, such as absolute- and relative-frequency bar graphs, histograms, stem and leaf plots, back-to-back stem and leaf plots, box plots, and scatter plots.

Students also learn to work with measures of central tendency, mean, mode, and median, understanding the attributes of each and how the measures change with the spread of the distributions. Computer programs and graphing calculators help students see how changes in the data affect the measures of central tendency.

An overriding goal of quantitative reasoning in grades 6-8 is the use of data to make observations, inferences, and conjectures. Students learn to formulate questions that arise from analysis of the data sets. Those questions can then be investigated with data analysis techniques to identify lack of relationship, linear and nonlinear relationships, and direct or inverse variation. Having formulated questions and investigated using appropriate techniques, students learn then to discuss what they have found in relation to the sample and whether what they have found can be generalized to a wider population or situation.

Middle level students need to experience investigations that lead them to think about chance and probability. Given many opportunities to experiment, students in grades 6-8 learn to analyze experimental outcomes, expected outcomes, and to contemplate any discrepancies that may arise. Making organized lists, tree diagrams, and area models, and using tools such as Pascal's Triangle, helps students learn to deal effectively with multiple outcomes for simple and compound events. Expected value explorations lead students to apply quantitative reasoning to real-world applications.

In the realm of real-world applications, students learn to be aware of data sources in newspapers, on television, in advertising. They then learn to realistically evaluate claims being made. Developing consumer awareness and savvy is a major goal of quantitative reasoning at the middle level.

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 1: Formulate questions and design statistical investigations that can be answered by data collection and analysis</p>	<p>Questions and conjectures</p>	<p>Formulate questions and design studies to collect data (categorical and numerical) about a characteristic shared by two populations or different characteristics within one population</p>		<p>Use data displays, including scatter-plots to make conjectures about possible relationships between two characteristics of a sample</p> <p>After formulating questions and collecting data, use conjectures to formulate new questions and design new statistical investigations to answer them</p>
	<p>Design for data collection</p>	<p>Distinguish between biased and unbiased forms of questions when designing investigations to collect valid data</p>		<p>Explore the influence of sample size and of random or not random sample selection; use these characterizations to evaluate the quality of collected data</p>
	<p>Sampling</p>	<p>Identify samples as representative or non-representative and articulate why a given sample is representative</p>	<p>Given a sample, use proportions to make estimates about a population based on that sample</p>	<p>Describe different methods of selecting statistical samples; analyze the strengths and weaknesses of each</p> <p>Distinguish between samples and populations; use the information drawn from samples to draw conclusions about populations</p> <p>Use observations about differences between two or more samples drawn from different populations to make conjectures about the populations from which the samples were taken</p>

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 2: Collect and display data with a variety of representations, justifying the appropriateness of methods used</p>	<p>Data collection and display</p>	<p>Collect, organize, and display data using a variety of representations (possible representations: bar graphs, line plots, scatter plots, frequency tables, stem-and-leaf plots, back-to-back stem-and-leaf plots, circle graphs)</p> <p>Evaluate the suitability of a particular data display for interpreting the given questions in a contextual situation</p>		<p>Collect, organize, and display data using a variety of representations including box plots</p>

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
<p>Learning Priority 3: Analyze data from organized displays and numerical descriptors; interpret information, make predictions, and draw conclusions with supporting arguments</p>	<p>Experimental Design</p>	<p>Evaluate and articulate possible misinterpretation of data and factors leading to faulty representations</p>		<p>Evaluate experimental design, statistical analysis, and reasonableness of conclusions from a variety of sources, including the media</p> <p>Describe the strengths and limitations of a particular statistical measure, and justify or critique its use in a given situation</p>
	<p>Measures of center, spread, and shape</p>	<p>Compute, describe and compare measures of center (mean and median), measures of spread (range) and shape (e.g., gaps, clusters, informal outliers), in real-world situations using technology when appropriate</p>		<p>Describe, interpret, and analyze representations of data sets using measures of spread (maximum, minimum, quartiles, inter-quartile range and statistical outliers), with and without graphing calculators</p>
	<p>Trend analysis</p>	<p>Interpret informal lines of best fit in approximately linear data and answer questions about trends and prediction, including the validity of the prediction</p>	<p>Interpret trends from displayed data; analyze the meaning of data points above and below the line of best fit in context</p> <p>In a relevant context use a given line of best fit to approximate the rate of change and to make predictions about values not in the original data set</p>	

Grades 6-8 Learning Continuum

Learning Priority	Focus Area	Grade 6	Grade 7	Grade 8
Learning Priority 4: Use a variety of counting techniques, experiments, and simulations to determine probabilities of events	Events	Understand and explain the concepts of equally likely and not equally likely events recognizing that fairness implies equally likely		
	Counting	Apply counting techniques to solve single and multi-step problems, justify solutions (e.g., tree diagrams, organized lists, frequency tables, area models, networks.)		
	Determining and using probabilities	Determine theoretical and experimental probabilities, distinguish the differences, and use these probabilities for prediction		Apply critical analyses to probability statements used to predict, make decisions, or answer questions, including media and real world sources

Draft.... for discussion and comment

Glossary Starter:

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|--|----------------------|------------|----------------------------|--------------------|
| Scale factor | Function | Part-whole | Part-part | Dynamic technology |
| Slope triangles | Ominoes | Mat plans | Congruence | Formal Outliers |
| Informal Outliers | Inter-quartile range | Planar | Undefined (divide by zero) | |
| Relevant contexts – relevant to student’s experience | | | | |