

## Revised WI Model Academic Standards for Mathematics – Grades 3-5

Instructional programs in grades three through five should address the three model academic standards—concepts and connections in number and algebra, concepts and connections in geometry and measurement, and concepts and connections in data analysis and probability. It is essential that these standards be embedded in contexts that promote and develop essential mathematical process skills—problem solving, conjecture, reasoning, and proof, representation and visualization, communication and reflection, and connections.

### PK-12 Standard: Concepts and Connections in Number and Algebra

#### Grades 3-5: Number and Operation Sense with Whole Numbers and Decimals (*Number, Operations, Algebra*)

- Learning Priority 1:** Develop understanding of base-ten concepts and equivalent representations, extend the base-ten numeration system to decimal numbers, and reason with multiplicative relationships
- Learning Priority 2:** Model and solve problems in context, develop and justify estimation and computation strategies, and be flexible in choosing strategies to find solutions
- Learning Priority 3:** Use algebraic reasoning to make conjectures about relations, properties, and operations, and to determine rules to describe functional relationships

#### Grades 3-5: Number and Operation Sense with Fractions (*Number, Operations, Algebra*)

- Learning Priority 1:** Develop understanding of the uses of fractions and the quantities they represent, and use models, benchmarks, and equivalent forms to compare and judge the size of fractions
- Learning Priority 2:** Develop meanings for operations with fractions in everyday situations, and develop, use, and evaluate strategies to estimate computations and to model and solve problems involving fractions

### PK-12 Standard: Concepts and Connections in Geometry and Measurement

#### Grades 3-5: Shape, Size, and Spatial Sense (*Geometry, Measurement, Number*)

- Learning Priority 1:** Visualize, describe, and reason about classes of shapes and investigate problems involving shapes, transformations, and spatial relationships
- Learning Priority 2:** Develop understanding of measurement concepts and attributes, and use and evaluate strategies to estimate and make measurements of familiar objects and aspects of our physical world

### PK-12 Standard: Concepts and Connections in Data Analysis and Probability

#### Grades 3-5: Describing and Analyzing Variability in Data Contexts (*Data Analysis, Number*)

- Learning Priority:** Formulate questions that anticipate variability of data in contextual situations, analyze and compare characteristics of data sets, and draw and justify conclusions in relation to the context

Students in grades 3-5 develop a broad range of strategies to solve problems. They model, represent, and solve single- and multi-step word problems involving a variety of combinations of the operations. They learn to flexibly choose an efficient strategy that is appropriate for the numbers in the problem, to make reasonable estimates, and to compute accurately. By comparing a variety of solution strategies, students develop an intuitive understanding of the basic properties of the operations and the relationships among them.

Students develop understanding of the meanings of multiplication and division and their inverse relationship through modeling contextual problems using a variety of representations (e.g., equal-sized groups, repeated addition, arrays, area models, and equal jumps on number lines for multiplication and successive subtraction, partitioning, and sharing for division). Students draw on their experiences with these models and representations to develop strategies for single-digit and multi-digit multiplication and division problems and use the strategies with fluency by the end of fifth grade.

Students apply their understanding of models for multiplication and division, place value, properties of operations, and relationships among operations to discuss, develop, and use flexible methods for estimating and computing with multi-digit numbers. Students select appropriate methods (e.g., mental calculations, calculators, algorithms) and use them to estimate and compute products and quotients.

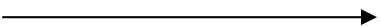
Students use drawings, graphs, numbers, equations, and manipulative models to represent their ideas and develop their understanding of mathematical concepts and relationships. They use these representations as they communicate mathematical arguments, justify solution paths, discuss connections among mathematical concepts, and interpret solutions to calculations (e.g., remainders in division problems). As they compare and contrast symbolic representations for situations they develop a deeper understanding of equality and use the equals sign to mean “represents the same quantity as” or “has the same value as” (e.g.,  $3+4=2+5$ ,  $3 \times 5=5+5+5$ ,  $3 \times 42=(3 \times 40)+(3 \times 2)$  ).

Students in these grades use models and other strategies to represent and study decimal numbers as they relate to fractions and to whole numbers. They come to understand decimal notation as an integral part of the base-ten system and as a way to represent rational numbers. Through a variety of activities, students understand that a fraction such as  $\frac{1}{2}$  is equivalent to  $\frac{5}{10}$ , that it has a decimal representation of 0.5, and that it is also equivalent to 0.50 and 0.500. As they investigate the relationship between fractions and decimals, they learn common fraction-decimal equivalents such as  $\frac{1}{4} = 0.25$ ,  $\frac{2}{5} = 0.4$ ,  $\frac{1}{2} = 0.5$ , and  $\frac{3}{4} = 0.75$ . Students relate their understanding of fractions to reading and writing decimals that are greater than or less than one, comparing and ordering numbers with decimals, and estimating decimal or fractional amounts in problem solving. They apply their understandings of decimal models, place value, and properties to add and subtract decimals. Another important understanding for students is the equivalence connection of whole numbers, fractions, and decimals through the meaning of a fraction as a quotient of two whole numbers. Here students learn that sharing 3 cookies among 4 people can be symbolized with fractions as  $\frac{3}{4}$ , as the division of two whole numbers,  $3 \div 4$ , or as a decimal, 0.75. Similarly, they understand these equivalences,  $\frac{7}{5} = 7 \div 5 = 1.4$ , and its connection to models and contexts such as 7 meters of string cut into five equal sections.

Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<p><b>Learning Priority 1:</b> Develop understanding of base-ten concepts and equivalent representations, extend the base-ten numeration system to decimal numbers, and reason with multiplicative relationships</p>	<p>Base-ten concepts and number magnitude</p>	<p>Show understanding of how a group of ten or one hundred or one thousand can be thought of as a single unit (e.g., 5 hundreds = 50 tens; 17 hundreds = 1 thousand 7 hundreds)</p> <p>Demonstrate understanding of the base-ten place value system and how the value of each place is 10 times that of the place to the right, and is one tenth the value of the place to the left</p> <p>Show understanding of place value by mentally adding and subtracting tens or hundreds from any number (e.g., <math>248 + 40 = 288</math>, <math>793 - 200 = 593</math>)</p> <p>Use place value ideas to compose and decompose multi-digit whole numbers in a variety of ways and to estimate the relative size of whole numbers (e.g., 386 is the same as 3 hundreds, 8 tens and 6 ones, or 2 hundreds, 17 tens and 16 ones; it's value is close to 400)</p>	<p>Show understanding of magnitude by counting forward and backwards by 0.1, 0.01, and by common fractional amounts (e.g., 0.25)</p> <p>Relate decimals and fractions as different notational systems for the same quantity and compare decimals to fractional benchmarks</p>	<p>Use place value ideas to compose and decompose decimals in a variety of ways and to estimate the relative size of decimals (e.g., 0.027 is the same as 2 hundredths and 7 thousandths or 27 thousandth; its value is a little less than 3 hundredths)</p>
	<p>Multiplicative relationships</p>	<p>Describe ways that numbers can be composed of equal groups (e.g., combining 3 groups of 8, 2 groups of 12, or 4 groups of 6 to get 24)</p> <p>Describe ways that numbers can be decomposed multiplicatively into factors (as opposed to additively), and use arrays, diagrams, and tables to show the meaning of the factors</p>		<p>Develop understanding of numbers as products of other numbers (e.g., Susan has 15 pencils or 5 times as many as Harold who has 3 pencils rather than additive reasoning that Susan has 12 more pencils.)</p>

Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<p><b>Learning Priority 2:</b> Model and solve problems in context, develop and justify estimation and computation strategies, and be flexible in choosing strategies to find solutions</p>	<p>Model, solve, and pose contextual problems</p>	<p>Model, solve, and pose contextual problems that involve one-step and multi-steps solutions. (Contexts should include a variety of situations, such as joining, separating, comparison, grouping, and partitioning.)</p>	<p>Model, solve, and pose single and multi-step contextual problems involving a variety of addition, subtraction, multiplication, and division situations (e.g., rate, measurement, comparison, arrays, areas, and combinations)</p>	<p>Interpret and justify remainders in multiple ways in contextual problems involving division</p>
	<p>Strategies for single-digit computation</p>	<p>Fluently use efficient and flexible strategies to solve single-digit addition and subtraction problems</p> <p>Develop and use strategies based on number properties (e.g., associative property, distributive property) to justify solutions to single-digit multiplication problems (e.g., double a known fact—<math>6 \times 4 = 2 \times (3 \times 4)</math>, use a relationship to five groups—<math>7 \times 6 = (5 + 2) \times 6 = (5 \times 6) + (2 \times 5) = 30 + 12</math>, compensate—<math>9 \times 4 = 10 \times 4 - 4</math>)</p>	<p>Develop and use the inverse relationship between multiplication and division to solve and justify solutions to division problems</p>	<p>Fluently use efficient and flexible strategies for single-digit multiplication problems and related division problems</p>

<p><b>Learning Priority 2:</b> Model and solve problems in context, develop and justify estimation and computation strategies, and be flexible in choosing strategies to find solutions</p>	<p>Flexible strategies for multi-digit computation and estimation</p>	<p>Develop, use, and justify flexible strategies to solve addition and subtraction problems, such as adding on in parts, compensating, or adding by place value. (Flexible refers to choosing an efficient strategy that is appropriate for the particular numbers in a problem.)</p> <p>Estimate solutions using base-ten understanding to judge the reasonableness of the results of computations.</p>	<p>Develop, use, and justify strategies to solve multiplication problems by using direct modeling with objects, skip counting, or repeated addition, and by using number relationships and place value knowledge with strategies such as doubling, partial products, or ratio tables.</p> <p>Develop fluency with computation and estimation strategies for multiplication, including choosing an efficient strategy that is appropriate for the particular numbers in the problem.</p>	<p>Develop, use, and justify strategies to solve division problems by using direct modeling such as partitioning using equal sharing or measuring using repeated addition or subtraction, and by using number relationships to subtract or build up groups of the divisor, or by using ratio tables.</p> <p>Develop fluency with computation and estimation strategies for division, including choosing an efficient strategy that is appropriate for the particular numbers in the problem.</p>
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Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<p><b>Learning Priority 3:</b> Use algebraic reasoning to make conjectures about relations, properties, and operations and to determine rules to describe functional relationships</p>	Equality	<p>Develop an understanding that the equal sign means “has the same value as.” </p> <p>Use properties, and relational thinking, rather than computation to reason about open number sentences (e.g., equations such as <math>7 + 8 = [ ] + 10</math>), and to determine and justify whether statements are true or false (e.g., Is <math>28 = 13 + 16</math> true or false? Is <math>57 + 44 = 56 + 45</math> true or false?)</p>		
	Properties and relationships of the operations	<p>Make conjectures about properties of the operations (e.g., associative and distributive properties) that emerge from solving problems and equations, and develop informal arguments to justify the conjectures.</p> <p>Develop, use, and explain relationships between addition, subtraction, multiplication, and division (e.g., multiplication can be thought of as repeated addition).</p>		
	Generalizing rules for functional relationships	<p>Recognize, extend, and generalize growing (functional) patterns found in physical, numeric, and geometric situations.</p> <p>Make conjectures about regularities in the number system (e.g., an even number plus and odd number is always odd.)</p>		

**Grades 3-5: Number and Operation Sense with Fractions** (*Number, Operations, Algebra*)

Students in grades 3-5 develop the foundations of number and operation sense for working with fractions. They develop understanding of the meanings and uses of fractions to represent parts of unit wholes, parts of a collection, points or distances on a number line, and divisions of whole numbers. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than one unit whole or multiple wholes. Students develop an understanding of equivalence in that amounts can have many names and they learn to rename fractions to higher and lower terms. They are encouraged to use the term “rename” rather than “reduce” or “simplify” to support conceptual understanding and extensions to other notational systems such as decimals and percents. Students use models, benchmarks, and equivalent forms to judge the size of fractions and to solve problems that involve comparing and ordering fractions.

Students apply their understandings of fractions and meanings of the operations to model addition and subtraction problem situations and make connections among real-world, concrete, pictorial, and symbolic representations. They develop flexible mental computation and estimation strategies for finding sums and differences of simple fractional amounts, wholes, and mixed numbers. Students begin with informal strategies for adding and subtracting fractions by using physical and visual models and benchmarks and progress to use of more formal procedures based on a solid understanding of the need to rename fractions to have the same sized parts prior to computing. Students also use physical models to explore simple everyday situations with fractions that involve combining equals groups, partitioning, and sharing and make connections among varied representations.

This foundation study of fractions focuses on the integration of fraction concepts and strategies, rather than the traditional approach to isolated ideas. From the first day of instruction, students should work with fractions that are greater than, less than, and equal to one whole, as well as include mixed numbers and whole numbers. Additionally, students should simultaneously study fractions with like and unlike denominators, as well mixed numbers and whole numbers, to avoid misconceptions and superficial strategies that do not generalize. Students need to develop flexible strategies for computing with fractions, such as being adaptable in how mixed numbers are named (e.g.,  $3 \frac{4}{5} = 2 \frac{9}{5}$ ), and be able to explain and justify their strategies, such as explaining when it is necessary to rename fractions when adding and when it is not needed.

Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<p><b>Learning Priority 1:</b> Develop understanding of the uses of fractions and the quantities they represent, and use models, benchmarks, and equivalent forms to compare and judge the size of fractions</p>	<p>Conceptual foundations</p>	<p>Develop understanding of fractions as parts of unit wholes in everyday contexts with emphasis on area models and partitioning situations (e.g., 4 brownies shared among 3 people)</p>	<p>Extend understanding of fractions as parts of unit wholes to interpretation of fractions as parts of a collection in everyday contexts</p>	<p>Extend understanding of fractions as divisions of whole numbers and include locations or distances on number lines</p>

<p><b>Learning Priority 1:</b>                  Develop understanding of the uses of fractions and the quantities they represent, and use models, benchmarks, and equivalent forms to compare and judge the size of fractions</p>	<p>Conceptual foundations</p>	<p>Connect oral language and symbols for fractions to the quantities they represent, using physical models and everyday objects</p>	<p>Relate, compose, and decompose fractions that are less than, equal to, and greater than one whole, including unit and non-unit fractions, and represent the quantities with language, objects, diagrams, and symbols</p>	<p>Represent and relate commonly used fractions, decimals, and percents as different notational systems for the same quantity</p>
	<p>Equivalence</p>	<p>Use oral language and models to recognize, explain, and show how different fractions “name” or represent the same amount, including fractions that are less than, equal to, and greater than one whole</p>	<p>Develop, use, and evaluate informal strategies for renaming fractions to larger terms (e.g., rename <math>\frac{3}{5}</math> as <math>\frac{9}{15}</math>, more parts in the unit whole) or to smaller terms (e.g., rename <math>\frac{6}{8}</math> as <math>\frac{3}{4}</math>, fewer parts in the unit whole)</p>	<p>Develop fluency in using a formal strategy for generating equivalent fractions and justify how the procedure is renaming the quantity without changing its value</p>
	<p>Relative magnitude</p>	<p>Use oral language and models to identify and explain whether a fraction is large or small relative to the size of the unit whole, and whether a fraction is more than, equal to, or less than one whole</p> <p>Use models and diagrams to compare two fractions situated in a familiar everyday context and determine which fraction represents the larger quantity</p>	<p>Use informal reasoning to identify and explain how far away a fraction is from one whole and whether a fraction is less than, greater than, or equal to one-half or one whole</p> <p>Develop and use informal strategies to compare two fractions to determine which is larger and which is smaller, including use of models, number lines, benchmarks, and equivalent forms</p>	<p>Use and evaluate strategies for comparing and ordering fractions</p>

Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<p><b>Learning Priority 2:</b> Develop meanings for operations with fractions in everyday situations, and develop, use, and evaluate strategies to estimate computations and to model and solve problems involving fractions</p>	<p>Meanings of the operations</p>	<p>Use visual models to explore, represent, and pose a variety of everyday problem situations involving addition and subtraction of fractions, and make connections among real-world, concrete, pictorial, and symbolic representations</p> <p>Extend understanding of the inverse relationship between addition and subtraction of fractions through models, language, and symbols</p>		<p>Model addition and subtraction situations and explore simple everyday situations involving multiplication and division of fractions, mixed numbers, and whole numbers</p> <p>Connect the language and symbolism of the operations with fractions to problems in context, such as posing word problems to match equations containing fractions</p> <p>Work with contextual problems to investigate and demonstrate relationships among operations with fractions</p>
	<p>Flexible estimation strategies</p>	<p>Estimate and justify whether the sum of two fractions is more than one whole or less than one whole in situations relevant to students' experiences</p>	<p>Develop, use, and justify strategies to estimate sums and differences of fractions and mixed numbers in situations relevant to students' experiences</p>	

<p><b>Learning Priority 2:</b> Develop meanings for operations with fractions in everyday situations, and develop, use, and evaluate strategies to estimate computations and to model and solve problems involving fractions</p>	<p>Flexible computation strategies</p>	<p>Use informal strategies for adding fractions that are less than, equal to, and greater than one whole with emphasis on language and physical models</p>	<p>Develop informal strategies using models, benchmarks, and equivalent forms for adding and subtracting fractions with emphasis on how adding fractions is similar to adding whole numbers in that “like units” are combined (e.g., fourths are added to fourths)</p> <p>Develop and explain mental computation strategies for finding the difference between whole numbers and fractional amounts (e.g., <math>3 - 1/4 = ?</math>)</p>	<p>Develop, use, and evaluate more formal strategies for addition and subtraction of fractions based on a solid understanding of the need to rename fractions to have the same sized parts (i.e., common denominators) prior to computing</p> <p>Use, explain, and evaluate flexible mental computation strategies for finding sums and differences of simple fractional amounts, whole numbers, and mixed numbers</p>
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**Grades 3-5: Shape, Size, and Spatial Sense** (*Geometry, Measurement, Number*)

Students in grades 3-5 recognize that geometric ideas and relationships can be applied to other mathematical strands, disciplines, and can be related to problems in everyday life. They compare and analyze characteristics and properties of two and three dimensional shapes and use these properties to define classes of polygons, polyhedra, and other three-dimensional solids (e.g., cylinders, spheres, and cones). They identify and construct both examples and non-examples of figures for specific classes of shapes and are able to explain and justify why a shape does or does not fit within that classification. Through building, drawing, and analyzing shapes in solving problems, students begin to develop ideas of how properties within shapes are related to each other, as well as how classes of shapes are related to each other. Students make and test conjectures about geometric properties and relationships and develop informal arguments to justify conclusions, such as explaining why a square is a special type of rectangle.

Students strengthen their spatial sense as they use visualization and spatial reasoning to solve problems and justify solutions. They create mental images of geometric shapes and relationships, and visualize shapes, objects, and structures from different perspectives. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. They predict and describe the results of translations (sliding), reflections (flipping), and rotations (turning) two-dimensional shapes and describe location, movement, and relative position. Students deepen their understanding of congruence and line and rotational symmetry as they apply transformations to analyze mathematical situations. Students within this grade band also identify and build three-dimensional objects from two-dimensional representations and draw two-dimensional representations of a three-dimensional object.

Students deepen their understanding of measurement concepts and skills as they form an understanding that measures of size are also attributes of two and three-dimensional objects. For example, students form an understanding of area as an attribute of two-dimensional regions and select appropriate units, strategies (e.g., decomposing shapes), and tools for solving problems that involve estimating or measuring area. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps and come to understand that the standard unit for measuring area is a unit square. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle. Extending these understandings to solids, students develop an understanding that a unit cube is the standard unit for measuring volume and decompose three-dimensional shapes as a strategy for solving problems that involve estimating or measuring volume.

Measurement experiences also allow students to further develop their measurement sense and understanding of standard measures by establishing personal benchmarks or references for common customary and metric units, such as a child knowing that the width of her hand is about 10 centimeters. Students use their personal benchmarks to make estimates, comparisons, and then verify values. Students also strengthen their understanding of fractions and decimals as they confront problems in linear measurement that call for more precision and require students to develop facility in measuring with fractional parts of linear units.

Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<p><b>Learning Priority 1:</b> Visualize, describe, and reason about classes of shapes and investigate problems involving shapes, transformations, and spatial relationships</p>	<p>Define classes of shapes by attributes</p>	<p>Analyze and classify polygons on the basis of attributes by side length, parallelism, and angle size, and use more precise ways to describe polygons</p> <p>Describe how to draw or build examples and non-examples of polygons in different sizes and orientations to represent the breadth of the class of shapes (e.g., draw obtuse and scalene triangles, build concave and convex quadrilaterals)</p> <p>Analyze, describe, and name attributes of polyhedra and other solids</p>		<p>Investigate polygons by lines of symmetry and diagonals and draw conclusions to formulate definitions of shapes. For example, “Can you make a quadrilateral with exactly two lines of symmetry? One line of symmetry? No lines? If so, in each case, what kind of quadrilateral is it?”</p> <p>Develop definitions of polygons and polyhedra that are based on properties and that distinguish classes of polygons and polyhedra</p>
	<p>Spatial visualization and</p>	<p>Use objects, drawings, and tools to investigate and describe the results of subdividing and combining polygons to make other polygons.</p>	<p>Visualize, create, and describe images of shapes, compositions, and decompositions of shapes (e.g., What results when we draw the diagonal of a rectangle?)g</p>	<p>Investigate how polygons can be decomposed into triangles, and draw conclusions about the area of a triangle by visualizing its relationship to a corresponding rectangle or other corresponding parallelogram</p>

<p><b>Learning Priority 1:</b> Visualize, describe, and reason about classes of shapes and investigate problems involving shapes, transformations, and spatial relationships</p>	<p>Spatial visualization and problem solving</p>	<p>Identify and build a three-dimensional object from a two-dimensional representation of that object</p>	<p>Investigate and reason about different nets (i.e., two-dimensional representations of the faces) for cubes and other polyhedra and justify why a net can or cannot be folded into specific polyhedra</p>	<p>Draw and describe two-dimensional representations that show different perspectives of a three-dimensional object</p> <p>Determine and provide justification for why a net can or cannot be folded into prisms by focusing attention to the number, shape, and relative positions of its faces</p>
	<p>Transformations</p>	<p>Investigate and describe three kinds of transformations or geometric motions— translations (slides), reflections (flips), and rotations (turns)</p>	<p>Visualize, predict, and demonstrate the results of geometric transformations</p>	<p>Visualize and describe the motion or a series of motions needed to prove congruence of a shape and its image (e.g., “reflect it vertically, then rotate it 180 degrees.”)</p> <p>Use precise language about turns and angles to describe the rotational symmetry of designs and objects, for example “If you turn it 180 degrees about the center, it’s exactly the same” or “It would take four equal small turns to get back to where you started”</p>

Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<p><b>Learning Priority 2:</b> Develop understanding of measurement concepts and attributes, and use and evaluate strategies to estimate and make measurements of familiar objects and aspects of our physical world</p>	<p>Measurement units and concepts</p>	<p>Demonstrate the need for measuring with standard units and become familiar with common units in the customary and metric systems by using them to estimate and measure attributes of familiar objects</p> <p>Demonstrate an informal understanding of how using smaller units or subdividing units increases precision of measurements (e.g., length, capacity, mass)</p> <p>Establish and use personal benchmarks to build meaning for customary and metric measures and to estimate measures in problem solving situations</p>		<p>Choose and justify appropriate selection of units for measuring specific attributes and objects</p> <p>Solve problems that require attention to both approximation and precision of measurement, and describe how measurements are never exact</p>

<p><b>Learning Priority 2:</b> Develop understanding of measurement concepts and attributes, and use and evaluate strategies to estimate and make measurements of familiar objects and aspects of our physical world</p>	<p>Measurable attributes and measurement strategies</p>	<p>Recognize perimeter as a measurable attribute of two-dimensional objects, and justify selection of appropriate measurement units, explain why the unit must be one-dimensional</p>	<p>Recognize area as a measurable attribute of two-dimensional regions, and justify selection of appropriate units (e.g., unit square), explaining why the unit must have two-dimensions</p>	<p>Recognize volume as a measurable attribute of space, and justify selection of appropriate units (e.g., unit cube), explaining why the unit must have three-dimensions</p>
		<p>Investigate and communicate strategies to solve problems that involve estimating or measuring perimeter of polygons and regions in the physical environment</p>	<p>Conjecture and communicate strategies, including decomposing, for solving problems that involve estimating or measuring area of two-dimensional shapes (regular and irregular) and regions in the physical environment</p>	<p>Estimate and justify volume of prisms by using a variety of strategies, including visualizing layers each composed of an array of cubes</p> <p>Describe what happens to measures of a two-dimensional shape when its perimeter or area is changed</p>
			<p>Use composition and decomposition of polygons to determine formulas for finding the area of rectangles and right triangles</p>	<p>Justify formulas for finding the area of parallelograms and triangles using relationships from decomposing and composing polygons</p>
		<p>Develop an understanding of right angles and their relationship to straight lines</p>	<p>Compare the relative size of angles by using informal notions of right angles (e.g., Is this angle larger or smaller than a right angle?)</p>	<p>Develop an understanding of angle size as a measurable attribute and use known angle sizes (e.g., right angle as 90 degrees) to estimate or determine the sizes of other angles</p>

**Grades 3-5: Describing and Analyzing Variability in Data Contexts** (*Data Analysis, Number*)

Students in grades 3-5 are inquisitive about their world and they realize that variability is inherent in all aspects of their lives. Individuals are different. People naturally have different heights, different interests and abilities, and different opinions. As students pose questions and explore data about themselves and their environment, they grow in their abilities to see a set of data as a whole and to consider the variation in the data along with key features as they informally draw reasonable conclusions and make predictions.

Students understand that data are generated with respect to particular contexts or situations and can then be used to answer questions about that context or situation. They formulate questions about a particular context, such as their class, and determine what data might be collected to answer these questions. As part of this process, they need to consider additional aspects of data collection such as: (1) how to refine the question in order to get the needed information, (2) whom to ask or what to observe, (3) what, when, and how to measure, and (4) how to record their data. Students have the option to collect their own data or use existing data sets to examine particular questions. If students collect their own data, they need to decide whether it is appropriate to conduct a survey or to use observations or measurements.

Students should view the purpose of analyzing the data as telling a story about the data as it relates to the context of the question. The use of data representations is to serve as a tool for analyzing data. Students should become familiar with a variety of representations such as tables, line plots, bar graphs, and line graphs. They compare representations and consider what new insights into the data emerge from different displays and evaluate how well important aspects of the data are communicated in the various displays. Students identify the nature of different kinds of data—categorical data and numerical data—and select and interpret appropriate representations. It is important to never allow the creation of data displays to overshadow the more essential focus on considering what the data story is telling us.

Noting the similarities and differences within one or between two related data sets requires students to become more precise in their descriptions of the data. Students notice important features of the data sets and how the data are spread across the range of values. For example, students notice where data are concentrated or clumped, identify values for which there are no data, or note data points that appear to have unusual values or are far removed from the rest of the data. Over these grades students gradually develop the idea of a “typical” or average value. Building on their informal understanding of “the most” and “the middle,” students develop an understanding of measures of center, such as median and mean. Students need to know more than simply how to identify the mode or median in a data set. They need to build an understanding of what, for example, the median tells them about the data, and they need to see this value in the context of other characteristics of the data. The concept of mean value—what it is, what information it gives about the data, and its interpretation in the context of other characteristics of the data—is a complex one. Students will explore the mean informally at this grade band, and it will be fully developed in the middle school grades.

With appropriate experiences, students begin to understand that many data sets are samples of larger populations. They can look at several samples drawn from the same population, such as different classrooms in their school, or compare information about their own sample to a larger population. Students begin to consider issues that affect the representativeness of a sample—how well it represents the population from which it is drawn—and begin to notice how samples from the same population can vary. Throughout these grades, students deepen their data sense, which is an understanding that data are more than just numbers—data are information about some context, and begin to consider the influence of variability on drawing conclusions and making predictions.

Learning Priority	Focus Area	Grade 3	Grade 4	Grade 5
<b>Learning Priority:</b> Formulate questions that anticipate variability of data in contextual situations, analyze and compare characteristics of data sets, and draw and justify conclusions in relation to the context	Formulate questions and collect data	<p>Formulate and refine questions arising from everyday situations and from other content areas that can be answered through the collection of data</p> <p>Determine what data, categorical or numeric, might be collected to answer questions and design a plan to collect and record the data (e.g., whom to ask, what to observe, what to measure)</p> <p>Make decisions on how to organize and display data (e.g., bar graphs, tables, and line plots) to interpret that data and to communicate that interpretation to others</p>	<p>Formulate and refine questions that reflect and anticipate variability in data, including questions that compare two groups, objects, or conditions</p> <p>Make conjectures and predictions regarding what the data might reveal about the context and then collect the data to answer the questions</p> <p>Compare different representations of the same data and evaluate how well each representation shows important aspects of the data</p>	
	Represent, analyze, and describe the data	<p>Describe observations of key data features and of the distribution of data points as it relates to the context of the original question</p>	<p>Develop a sense of how the center, spread, and distribution of the data points are useful in describing the data context (e.g., where the data are concentrated or spread out, what are the lowest and highest values)</p> <p>Develop an understanding of median and range as important features for comparing related sets</p>	
	Draw and justify conclusions from data	<p>Interpret various single data sets (existing or collected), draw conclusions, and summarize results related to the original question</p>	<p>Summarize the data in two related data sets and draw and justify conclusions in relation to the question and context.</p> <p>Recognize how variability in data affects the conclusions that can be drawn.</p> <p>Examine and describe representativeness of data (e.g., Is this true for all fifth graders in our class? Our school? Our State?).</p>	